Dependability analysis of a very large volume neutrino telescope

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ABSTRACT

This work considers a first order approximation to the dependability analysis of complex large scale installations. The dependability criterion used here is quantitative unavailability, and an appropriate unavailability model is presented. The model assumes that the system is symmetrical, has various levels of hierarchy, and components found in the same level are similar and function independently.

The application example comes from very large volume neutrino telescopes installed under water or ice, consisting of several thousands of optical modules. The readout architecture of the detector has several levels of multiplexing including optical detection towers, branches and tower sectors. The paper presents results for various alternative detector layouts and distances of the detector from the onshore facilities. It also develops dependability requirements for major components and/or subsystems consistent with an overall system performance target. The results depict the dependence of the system unavailability on the number of optical modules and the alternative deep sea infrastructure configurations for transferring the measured signals.

Keywords: large neutrino telescope, reliability, availability allocation

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1. INTRODUCTION

Astronomical objects can be observed using Cherenkov telescopes to detect the high-energy astrophysical neutrinos they emit. In Cherenkov telescopes water or ice has the dual role of absorbing daylight and the vast majority of charged cosmic rays, while allowing the detection of atmospheric or astrophysical neutrinos through the Cherenkov light emitted by muonic reaction products in aqueous transparent media ^[1]. These telescopes employ a grid of several thousands of Optical Modules (OM) to measure this light and enable optical neutrino detection. According to solid theoretical arguments, the total volume of the detector grid should be at least of the scale of a cubic kilometer, as in the cases of IceCube ^[2] which is expected to be complete in 2011 and KM3NeT ^[3] which is in preparatory phase ^[4]. The OMs consist of the photon sensors and the associated electronics housed in a glass sphere. To form a spatial grid, the OMs are placed on mechanical structures, the Detection Units (DUs), supporting and connecting them in vertical assemblies. The OMs are interconnected via watertight and pressure-resistant connectors, and their measured data travel through an underwater network of multiplexed passive optical or active electronic equipment and optical fibers to the onshore base ^[5].

Dependability is the ability of a system to perform its *required function*, under determined operating conditions, during a given period of operation. The dependability measure used here is the overall telescope *unavailability*, namely the percentage of time that the telescope is expected to be unavailable given the desired period of operation. The telescope is treated as a complex system; therefore its unavailability can be expressed as a function of the unavailabilities of the telescope components. The following sections discuss the development of an appropriate model to determine: (a) the system unavailability given the unavailabilities of its components, and (b) the component unavailabilities that satisfy a given performance criterion.

The paper is organized as follows. Section 2 describes the main relevant characteristics of the neutrino telescope. Section 3 presents the method of analysis and the model linking the component unavailabilities with the system unavailability. Section 4 presents and discusses some numerical results, and section 5 concludes this work.

2. SYSTEM DESCRIPTION

The large volume neutrino telescope consists of various components and subsystems that can be distinguished into three major subsystems: the detector network, the deep sea infrastructure and the onshore infrastructure ^[5] (see Figure 1). All three subsystems must be successful for the telescope to be successful. This work considers the dependability analysis of the OM network and the deep-sea infrastructure. Depending on the specific design, the components belonging to each of the above subsystems and their interconnection layouts may differ. This section provides a conceptual description of the telescope's detector grid and cabling.

Figure 1: Overall neutrino telescope system

The Detector Network consists of the OMs grouped to form DUs, the DUs grouped into branches, and branches into sectors, as described below. The OM block contains a multiple photomultiplier tube (PMT), the associated electronics and all the cabling connecting it with the Junction Box (JB) of the DU. At the onshore base the data will be processed, stored and analyzed further to find hit patterns corresponding to neutrino-generated muon tracks producing light in the water ^[8].

The readout architecture of the detector has several levels of multiplexing, starting from the glass sphere connector where the analogue PMT electrical signal is digitized. At each DU, the digitized outputs of the optical modules are gathered in the JB of the DU. This JB contains all the necessary electronics to process the signals from the OMs and the necessary cabling for the signals to travel to shore. Operability of a DU is not defined in terms of the number of the available OM since, as mentioned previously, only the total number of available OMs in the system is of importance. A DU is unavailable if there is a structural failure and the DU collapses, or when the JB of the DU fails, or when the connection of the JB to the DU sector cable fails. The output signals of the DU JBs travel through a cable network to the Primary Junction Box(es) (PJB(s)). Apart from optical fibers, all the network cables include electrical wires, to provide the power supply necessary for the operation of the telescope. The DUs are placed on the seabed forming a rectangular layout. Figure 2 gives a 3D ground plan view of the rectangular arrangement of the DU grid on the seabed assuming a set of two PJBs operating in parallel. On each horizontal string of the grid, the DUs are connected on the same

cable, starting from the PJB1 and finishing in PJB2. The cable contains optical fibers and each fiber can serve more than one DU, as Figure 2 illustrates. DUs are grouped into sectors according to the cable they are connected to, and sector branches according to the cable fiber they are connected to. A branch is unavailable when the fiber connecting the DUs is unavailable. Likewise, a sector is unavailable when the sector cable connecting the two primary junction boxes is unavailable. The Deep-Sea Infrastructure consists of the PJB(s) and one main cable carrying the output signal of the PJB(s) to the onshore base.

Figure 2: Three-dimensional detector network layout

In dependability analysis it is essential to determine the *required function* of the system under consideration. The objective of the neutrino telescope is to record and analyze neutrino interactions with matter. Simulation experiments with alternative geometries of the conceptual design have established that the telescope performance depends mainly on the number of available OMs and not on their spatial distribution (within a given number of available geometries) ^[5–7]. For this reason it is reasonable to assume here that: *the required function of the telescope is to have an appropriate number of OMs operable and available to detect neutrino interactions and to transfer their measured data back to shore.*

3. UNAVAILABILITY MODEL

The development of the DN unavailability model is based on (a) the network component failure assumptions discussed in the previous section, (b) a success criterion based on the minimum number of available OMs and (c) a set of design assumptions described bellow. It is

also assumed that all the system components can be in one of the two states: available or unavailable to perform their required operations.

The functional relationship of the various parts of the DN is developed as a hierarchical structure made up of four layers of components: the top layer is the sector cables, then the sector cable fibers (or branches) and the DUs, and finally the OM's as the lowest level layer. Failure of a top layer component renders unavailable all the lower level components associated with the failed component. Therefore, when a sector component fails all the branches, DUs and OM's on the sector become unavailable. Similarly, all the DUs and OM's of a failed branch are unavailable, and all the OM's of a failed DU are unavailable. However, component failures do not affect other components on the same or higher layers. Another important assumption is that all the same layer components are identical, meaning that they have the same reliability, maintenance and repair features, and that they are connected to the same number of lower layer(s) components. Therefore, all the sectors, branches and DUs are connected to the same number of branches, DUs and OM's, respectively. Note that, all the assumptions are in good agreement with the generic features of the KMeNeT telescope as it currently being designed ^[7].

The overall logical interconnection of the system components is given in Figure 3.

Figure 3: Detector network block diagrams

3.1. General Considerations

The unavailability of the DN has been calculated as a function of the unavailability of its components under the following assumption: The unavailability of each and every component is assumed constant and equal to the average unavailability over the period of analysis (lifetime of the telescope). As a result of this assumption, the unavailability of the system is also constant and it can be calculated through static reliability analysis. This a first approximation of the systems operating behavior and it aims at identifying a first approximation of the order of magnitude of the various dependability characteristics of the components. These in turn, will be set as design and manufacturing targets for the various components of the system. Depending on the feasibility of achieving these dependability goals by the various components, the final design (i.e. number of OMs and overall size of the telescope) will be determined.

Recursive formulae are developed to express the unavailability of the system in terms of the unavailability of the components in each layer. In this part of the analysis no assumption is made about the nature of the failure and/or repair rates of the components, or the testing and repair methods. In the remaining of this subsection, a number of general relationships used in the unavailability derivation are presented.

Let N_k be the number of components of type k in the DN, $k \in \{1,2,3,4\}$, where indices 1, 2, 3 and 4 denote sectors, branches, DUs and OMs, respectively. Let n_k be the number of components of layer k connected to each component of layer $k-1$, and $n_1=N_1$. Then

$$N_k = N_d \cdot \prod_{i=d+1}^k n_i, \quad \forall d \leq k \quad (1)$$

Let K_{OM} denote the minimum number of available OMs for the telescope system to be available. Then the maximum percentage, c , of OMs that if failed the system is still considered available is a known quantity given as $c \equiv (N_4 - K_{OM})/N_4$.

The development of the DN model follows the system hierarchy described above, starting from the top layer and proceeding to the lower layers. Let m_k denote the number of failed layer k components, and m_i^* be the maximum m_k for the system to be available.

Since failures of same layer components occur independently, the probability of having a certain number of failed components on layer k is calculated as follows. Consider the set of the mutually exclusive events

$$X_k = 0, X_k = 1, \dots, X_k = m_k, \text{ and } X_k > m_k$$

The probability of up to and including m_k components being unavailable is given by ^[9]

$$\Pr[X_k \leq m_k] = \sum_{i_k=0}^{m_k} \Pr[X_k = i_k] \quad (2)$$

Assuming that the events X_k follow the binomial probability distribution, for a sample of N_k identical components with probability p_k for each component to be unavailable

$$P[X_k = i_k] = \binom{N_k}{i_k} p_k^{i_k} \cdot (1 - p_k)^{N_k - i_k}$$

and equation (2) becomes

$$\Pr[X_k \leq m_k] = \sum_{i_k=0}^{m_k} \binom{N_k}{i_k} p_k^{i_k} \cdot (1 - p_k)^{N_k - i_k} \quad (3)$$

The probability of over m_k components being unavailable is

$$\Pr[X_k > m_k] = 1 - \sum_{i_k=0}^{m_k} \binom{N_k}{i_k} p_k^{i_k} \cdot (1 - p_k)^{N_k - i_k} \quad (4)$$

The following subsections describe the development of the detector model starting from the top layer components (sectors), down to the lowest layer (OMs).

3.2. Layer 1: Sector component

Let X_1 denote the number of unavailable sectors and m_1^* be the maximum number of unavailable sectors for the system to be operational (see Figure 3). Then

$$m_1^* = \text{int}[c \cdot N_1] = \text{int}[c \cdot n_1] \quad (5)$$

Let DN denote the event “Detector Network unavailable” and U_{DN} denote the probability of event DN . For $m_1 = m_1^*$, consider the complete set of the mutually exclusive events

$$X_1 = 0, X_1 = 1, \dots, X_1 = m_1, \text{ and } X_1 > m_1$$

Then the U_{DN} can be written as

$$U_{DN} = \sum_{i_1=0}^{m_1} (\Pr[X_1 = i_1, DN]) + \Pr[X_1 > m_1, DN] \quad (6)$$

where, $\Pr[X_1 = i_1, DN]$ denotes the probability of the joint event that the system is unavailable and i_1 sectors are unavailable, while $\Pr[X_1 > m_1, DN]$ denotes the probability of the joint event that the system is unavailable and more than m_1 sectors are unavailable.

Expansion of the joint event probabilities in Equation (6) yields

$$U_{DN} = \sum_{i_1=0}^{m_1} \Pr[X_1 = i_1] \cdot U(DN|X_1 = i_1) + \Pr[X_1 > m_1] \cdot U(DN|X_1 > m_1) \quad (7)$$

where $U(DN|X_1 = i_1)$ is the unavailability of the system given that i_1 sectors are unavailable, and $U(DN|X_1 > m_1)$ is the unavailability of the system given that more than m_1 sectors are unavailable.

If more than m_1 sectors are unavailable, then the criterion for at least K_{OM} available OMs is violated and the DN is unavailable, hence $U(DN|X_1 > m_1) = 1$ and equation (7) becomes

$$U_{DN} = \sum_{i_1=0}^{m_1} \left(\Pr[X_1 = i_1] \cdot U(DN|X_1 = i_1) \right) + \Pr[X_1 > m_1] \quad (8)$$

Assuming that X_1 follows the binomial probability distribution, for a sample of n_1 sectors and probability p_1 for each sector to be unavailable

$$\Pr[X_1 = i_1] = \binom{n_1}{i_1} \cdot p_1^{i_1} \cdot (1 - p_1)^{n_1 - i_1} \quad (9)$$

and

$$\Pr[X_1 > m_1] = 1 - \sum_{i_1=0}^{m_1} \Pr[X_1 = i_1] \quad (10)$$

3.3. Layer 2: Branch component

Let m_2^* be the maximum number of unavailable branches with which the system is considered operating (see Figure 3). Based on equation (1)

$$m_2^* = \text{int}[c \cdot N_2] = \text{int}[c \cdot n_1 \cdot n_2] \quad (11)$$

However, when i_1 sectors are unavailable $i_1 \cdot n_2$ branches are also unavailable. The maximum number of branches that can be unavailable without violating the success criterion is, therefore, reduced to

$$m_2(i_1) = m_2^* - i_1 \cdot n_2 \quad (12)$$

Consider the complete set of mutually exclusive events

$$X_2 = 0, X_2 = 1, \dots, X_2 = m_2(i_1), \text{ and } X_2 > m_2(i_1) \quad (13)$$

Then the term $U(DN|X_1 = i_1)$ of equation (8) can then be written as

$$U(DN|X_1 = i_1) = \sum_{i_2=0}^{m_2(i_1)} \left(\Pr[X_2 = i_2|X_1 = i_1] \cdot U(DN|X_1 = i_1, X_2 = i_2) \right) + \Pr[X_2 > m_2(i_1)|X_1 = i_1] \cdot U(DN|X_1 = i_1, X_2 > m_2(i_1)) \quad (14)$$

where i_2 denotes the number of unavailable branches.

Let the number of the remaining branches when i_1 sectors are down be denoted by $n_2^*(i_1)$ and given by

$$n_2^*(i_1) = (n_1 - i_1) \cdot n_2 \quad (15)$$

If more than $m_2(i_1)$ branches become unavailable, the criterion for at least K_{OM} available OMs is violated and the DN is unavailable, thus $U(DN|X_1 = i_1, X_2 > n_2) = 1$ and equation (14) writes

$$U(DN|i_1) = \sum_{i_2=0}^{m_2(i_1)} \Pr[X_2 = i_2|i_1] \cdot U(DN|i_1, i_2) + \Pr[X_2 > m_2(i_1)|i_1] \quad (16)$$

where the events $\{X_k = i_k\}$ have been simplified to $\{i_k\}$.

Assuming that X_2 follows the binomial distribution, for a sample of $n_2^*(i_1)$ branches and probability p_2 for each branch to be unavailable

$$\Pr[X_2 = i_2|X_1 = i_1] = \binom{n_2^*(i_1)}{i_2} \cdot p_2^{i_2} \cdot (1 - p_2)^{n_2^*(i_1) - i_2} \quad (17)$$

and

$$\Pr[X_2 > m_2(i_1)|X_1 = i_1] = 1 - \sum_{i_2=0}^{m_2(i_1)} \Pr[X_2 = i_2|X_1 = i_1] \quad (18)$$

3.4. Layer 3: Detection unit component

Let now m_3^* denote the maximum number of unavailable DUs with which the system is considered operational (see Figure 3). Based on equation (1)

$$m_3^* = \text{int}[c \cdot N_3] = \text{int}[c \cdot n_1 \cdot n_2 \cdot n_3] \quad (19)$$

When i_1 sectors are unavailable and i_2 additional branches (beyond those belonging to the i_1 sectors) are unavailable, the maximum number of extra DUs that can be unavailable is

$$m_3(i_1, i_2) = m_3^* - (i_1 \cdot n_2 + i_2) \cdot n_3 \quad (20)$$

Consider the complete set of mutually exclusive events

$$X_3 = 0, X_3 = 1, \dots, X_3 = m_3(i_1, i_2), \text{ and } X_3 > m_3(i_1, i_2)$$

Then the term $U(DN|i_1, i_2)$ of equation (16) can be written as

$$U(DN|i_1, i_2) = \sum_{i_3=0}^{m_3(i_1, i_2)} \left(\Pr[X_3 = i_3|i_1, i_2] \cdot U(DN|i_1, i_2, i_3) \right) + \Pr[X_3 > m_3(i_1, i_2)|i_1, i_2] \cdot U(DN|i_1, i_2, X_3 > m_3(i_1, i_2)) \quad (21)$$

where i_3 denotes the number of unavailable DUs.

The number of DUs outside those in the failed i_1 sectors and i_2 branches is

$$n_3^*(i_1, i_2) = (n_2^*(i_1) - i_2) \cdot n_3 \quad (22)$$

If more than $m_3(i_1, i_2)$ DUs become unavailable, the criterion for at least K_{OM} available OMs is violated and the DN is unavailable, thus $U(DN|i_1, i_2, X_3 > n_3(i_1, i_2)) = 1$ and equation (21) becomes

$$U(DN|i_1, i_2) = \sum_{i_3=0}^{n_3^*(i_1, i_2)} \left(\Pr[X_3 = i_3|i_1, i_2] \cdot U(DN|i_1, i_2, i_3) \right) + \Pr[X_3 > m_3(i_1, i_2)|i_1, i_2] \quad (23)$$

Assuming that X_3 follows the binomial probability distribution, for a sample of $n_3^*(i_1, i_2)$ DUs and probability p_3 for each DU to be unavailable

$$\Pr[X_3 = i_3|i_1, i_2] = \binom{n_3^*(i_1, i_2)}{i_3} \cdot p_3^{i_3} \cdot (1 - p_3)^{n_3^*(i_1, i_2) - i_3} \quad (24)$$

and

$$\Pr[X_3 > m_3(i_1, i_2)|i_1, i_2] = 1 - \sum_{i_3=0}^{m_3(i_1, i_2)} \Pr[X_3 = i_3|i_1, i_2] \quad (25)$$

3.5. Layer 4: Optical module component

Let now m_4^* denote the maximum number of unavailable OMs with which the system is considered operational (see Figure 3). Based on equation (1)

$$m_4^* = \text{int}[c \cdot N_4] = \text{int}[c \cdot n_1 \cdot n_2 \cdot n_3 \cdot n_4] \quad (26)$$

When i_1 sectors, i_2 branches and i_3 DUs are unavailable, the maximum number of extra OMs that can be unavailable for the DN to be available is reduced to

$$m_4(i_1, i_2, i_3) = m_4^* - [(i_1 \cdot n_2 + i_2) \cdot n_3 + i_3] \cdot n_4 \quad (27)$$

Consider the complete set of the mutually exclusive events

$$X_4 = 0, X_4 = 1, X_4 = 2, \dots, X_4 = m_4(i_1, i_2, i_3), \text{ and } X_4 > m_4(i_1, i_2, i_3)$$

Then the term $U(DN|i_1, i_2, i_3)$ of equation (23) can be written as

$$U(DN|i_1, i_2, i_3) = \sum_{i_4=0}^{m_4(i_1, i_2, i_3)} \left(\Pr[X_4 = i_4 | i_1, i_2, i_3] \cdot U(DN|i_1, i_2, i_3, i_4) \right) + \Pr[X_4 > m_4(i_1, i_2, i_3)] \cdot U(DN|i_1, i_2, i_3, X_4 > m_4(i_1, i_2, i_3)) \quad (28)$$

where i_4 denotes the number of unavailable OMs.

If more than $m_4(i_1, i_2, i_3)$ OMs become unavailable, the criterion for at least K_{OM} available OMs is violated and the DN is unavailable, thus $U(DN|i_1, i_2, i_3, X_4 > n_4(i_1, i_2, i_3)) = 1$. Additionally, m_4^* is known and equal to $(N_4 - K_{OM})$, therefore, the DN is always available for any $i_4 \leq m_4(i_1, i_2, i_3)$, thus $U(DN|i_1, i_2, i_3, i_4) = 0$. Finally, equation (28) becomes

$$U(DN|i_1, i_2, i_3) = \Pr[X_4 > n_4(i_1, i_2, i_3)] \quad (29)$$

The total number of remaining OMs, outside the i_1 unavailable sectors, the i_2 unavailable branches and the i_3 unavailable DUs is

$$n_4^*(i_1, i_2, i_3) = (n_3^*(i_1, i_2) - i_3) \cdot n_4 \quad (30)$$

Using equation (30), equation (29) becomes

$$U(DN|i_1, i_2, i_3) = 1 - \sum_{i_4=0}^{n_4^*(i_1, i_2, i_3)} \Pr[X_4 = i_4 | i_1, i_2, i_3] \quad (31)$$

Assuming that X_4 follows the binomial probability distribution, for a sample of $n_4^*(i_1, i_2, i_3)$ OMs and probability p_4 for each OM to be unavailable

$$\Pr[X_4 = i_4 | i_1, i_2, i_3] = \binom{n_4^*(i_1, i_2, i_3)}{i_4} \cdot p_4^{i_4} \cdot (1 - p_4)^{n_4^*(i_1, i_2, i_3) - i_4} \quad (32)$$

3.6. System unavailability

It follows that the DN unavailability can be calculated recursively from equations (31), (23), (16) and (8), assuming that the unavailabilities of each system component are known. Note that, the model developed here for the DN can easily be extended to systems of more than 4 layers.

Once U_{DN} is determined, the overall unavailability U_T of the serial system composed of the OM network and the deep sea infrastructure is evaluated as

$$U_T = U_{DN} + U_0 - U_{DN} \cdot U_0 \quad (33)$$

where U_0 is the combined unavailability of the PJB(s) and the cable to shore. Since these

are connected in series, U_0 is given by

$$U_0 = p_c + p_0 - p_c \cdot p_0 \quad (34)$$

where, p_0 and p_c denote the unavailabilities of the PJB(s) and the main cable components, respectively.

4. COMPONENT AVAILABILITY ALLOCATION

The objective of the dependability analysis presented in this paper is to obtain a first order approximation of the required component availability characteristics in order to achieve an overall system availability requirement.

The base case design considered here is a cuboid grid of $N_4 = 17 \times 18 \times 20 = 6120$ OM s. The OM s are arranged in $N_3 = 306$ DU s each containing 20 OM s. The DU s are grouped into $N_2 = 102$ branches each containing 3 DU s. The branches are grouped into $N_1 = 17$ sectors each containing 6 branches. This configuration has $n_1 = 17$, $n_2 = 6$, $n_3 = 3$ and $n_4 = 20$ and represented by the vector [17, 6, 3, 20].

Since the performance for the telescope depends on the number of available OM regardless of their spatial distribution ^[5-7], the OM s can be treated as completely identical and equivalent components regardless of their position in the telescope, and the analysis of section 3 becomes applicable. For the basic design of the OM s and the general configuration of the neutrino telescope, the following availability criteria have been established

- Mission time of the system, T , is 10 years.
- The minimum number of available OM s for the telescope to operate successfully is set at 90% of the OM s in the base design, thus $K_{OM} = 5508$ available OM s.
- For at least 90% of the mission time more than K_{OM} OM s must be available to detect neutrino interactions and transmit the corresponding information to the shore, namely $U_T \leq 10^{-1}$.

4.1. Component maintenance

One of the telescope design issues is the layout of the OM s into different configurations, in order to achieve minimum cost and maximum availability. Additional issues include the required reliability characteristics of the various components, so that the overall system dependability performance requirement is met. This section further explores these issues under the assumption that components are characterized by constant failure and repair rates.

Two factors determine the component availability

- a) the component's mean-time-to-failure (MTTF) and
- b) whether the component is repairable and if yes its mean-time-to-repair (MTTR)

The same average unavailability can be achieved by a non-repairable component with long MTTF or by a repairable component with a shorter MTTF and an appropriate MTTR. An important decision in the design of the telescope is, therefore, whether the layout would be such to allow the retrievability of the various components and subsystems. At this stage of the telescope's development it appears that the OMs and the DUs won't be retrievable, for a number of technical and cost-related reasons.

In the system unavailability calculations the relationships developed in section 3 are used. Average, constant unavailability values for each component are assumed in the place of p_1 to p_4 in equations (8) to (31). This is an acceptable approximation, provided that the average or the steady-state component unavailabilities are obtained in relatively short times with respect to the system's mission time. Let $\lambda=1/MTTF$ denote the component failure rate.

For non-repairable components operating over a period of time T , the average unavailability relationship is according to Rausand and Hoyland ^[10]

$$\bar{p}_a = 1 - \frac{1}{\lambda T} \cdot (1 - e^{-\lambda T}) \quad (35)$$

For repairable components where the failure is detected immediately and the repair starts also immediately with $d=MTTR$, the steady state unavailability is given by Kumamoto and Henley ^[11] as

$$\bar{p}_b = \frac{d}{MTTF + d} \quad (36)$$

For repairable components, testable or reachable every T_o units of time and with constant repair duration d , the average unavailability is given by

$$\bar{p}_{c,int}(\infty) = 1 - \frac{1 - e^{-\lambda(T_o+d)}}{\lambda(T_o+d)(1 + e^{-\lambda T_o} - e^{-\lambda(T_o+d)})} \quad (37)$$

Proof of equation (37) is provided in the Appendix.

4.2. Unavailability results

This section examines a variety of OM network layouts and evaluates feasible sets of component unavailabilities for the telescope to achieve a given unavailability target. As mentioned above, the overall maximum telescope unavailability U_T used here is set at 10^{-1} .

In each unavailability estimation, given are the total number of optical modules and their

network arrangement into DUs, branches, and sectors. Seven configurations are considered, all having the same DU structure ($n_4=20$) and type of sector cables ($n_2=3$). These are the [17,6,3,20], [18,6,3,20], [19,6,3,20], [20,6,3,20], [19,7,3,20], [20,7,3,20] and [21,7,3,20], corresponding to 6120 to 8820 OMs.

The following assumptions are made

- The unavailability of the main cable p_c ranges from 10^{-3} to 8×10^{-2} . If the cable is reachable and immediately repairable with one month MTTR, these unavailabilities translate to MTTFs from 80 to 1 years, respectively, according to equation (36).
- The unavailability of the PJB(s) p_0 is fixed at 2×10^{-2} . If the PJB(s) is(are) reachable once a year and repaired for one month, the corresponding MTTF is 30 years, according to equation (37).
- The unavailability of the sector p_1 ranges from 5×10^{-3} to 5×10^{-2} . The corresponding MTTFs for a non-repairable component featuring these unavailabilities are 1000 to 100 years, respectively, according to equation (35).
- The unavailability of the branch is always $p_2 = 5 \cdot p_1$, meaning that an optical fiber is assumed five times more likely to fail than the cable containing it. Their unavailability ranges from 2.5×10^{-2} to 2.5×10^{-1} , giving non-repairable component MTTFs from 200 to 16.5 years, respectively, according to equation (35).
- The unavailability of the detection unit p_3 ranges from 7×10^{-3} to 3×10^{-1} , yielding MTTFs for non-repairable components from 700 to 13 years, respectively, according to equation (35).
- The unavailability of the optical module p_4 ranges from 7×10^{-4} to 3×10^{-1} , yielding MTTFs for non-repairable components from 7000 to 13 years, respectively, according to equation (35).

Since U_T is fixed, the model of section 3 is solved to provide combinations of unavailabilities of the main cable, the PJB(s), the sectors, the branches, the DUs and the OMs, which satisfy this system unavailability value. Figures 4 to 8 present indicative results for each network layout.

Figure 4 presents the results for the [17, 6, 3, 20] configuration. For this configuration results within the assumed unavailability ranges exist only if the unavailability of the sector (p_1) is 5×10^{-3} or less. For non-repairable components this implies a $\text{MTTF} \geq 1000$ years. If the $p_1 = 5 \times 10^{-3}$ unavailability is achievable then the maximum main cable unavailability (p_c) is 6×10^{-2} and the corresponding DU and OM unavailabilities are $p_3 = 7.5 \times 10^{-3}$ and

Figure 4: Component unavailabilities for the detector configuration [17, 6, 3, 20]

Figure 5: Component unavailabilities for the detector configuration [18, 6, 3, 20]

Figure 6: Component unavailabilities for the detector configuration [19, 6, 3, 20]

$p_4 = 1.6 \times 10^{-3}$, respectively. If $p_c = 10^{-3}$ the possible combinations of p_3, p_4 unavailabilities are given by the upper limit curve (solid line) in Figure 4. Likewise, for $p_c = 6 \times 10^{-2}$ the possible p_3, p_4 combinations are given by the lower limit curve (dashed line) in Figure 4. The gray area indicates the possible values of the triplets (p_c, p_3, p_4) . Higher sector unavailability values violate the constraint on U_T .

Figure 5 presents the results for the [18, 6, 3, 20] configuration. For this configuration results within the assumed unavailability ranges exist only if the sector unavailability (p_1) is less than 10^{-2} . If $p_1 = 5 \times 10^{-3}$ the cable unavailability can vary between 8×10^{-2} and 10^{-3} , and the range of possible (p_3, p_4) values, for the DU and the OM unavailabilities, is given by the gray area. For $p_1 = 5 \times 10^{-3}$ and $p_c = 10^{-3}$ the possible combinations of p_3, p_4 unavailabilities are given by the upper limit curve (solid line, bold for $p_c = 10^{-3}$). For $p_1 = 5 \times 10^{-3}$ and $p_c = 8 \times 10^{-2}$ the possible combinations of p_3, p_4 unavailabilities are given by the lower limit curve (dashed line, bold for $p_c = 10^{-3}$). If p_1 increases to 10^{-2} then lower values for the unavailabilities of the other components are required. The possible values for the cable unavailability are now between 10^{-3} and 7×10^{-2} , while the corresponding p_3, p_4 unavailabilities are contained in the white/blue upward diagonal area in Figure 5.

Figure 6 presents the results for the [19, 6, 3, 20] configuration. For this configuration results within the assumed unavailability ranges exist only for p_1 less than 2×10^{-2} . For $p_1 = 5 \times 10^{-3}$ the cable unavailability p_c lies between 8×10^{-2} and 10^{-3} and the range of possible values for p_3 and p_4 is given by the gray area. Higher p_1 values call for lower unavailabilities of the other components. The white/blue upward diagonal area, the orange with bold limiting curves and the white/purple downward diagonal shaded areas correspond to p_1 values of 10^{-2} , 1.5×10^{-2} and 2×10^{-2} , respectively.

Figure 7 presents the results for the [20, 6, 3, 20] configuration. For this configuration results within the assumed unavailability ranges exist only for p_1 less than 2.5×10^{-2} . For $p_c = 8 \times 10^{-2}$, p_3 drops from 2×10^{-1} to 4×10^{-2} as p_1 increases from 5×10^{-3} to 2.5×10^{-2} , respectively. The green area with bold limiting curves corresponds here to p_1 values of 2.5×10^{-2} .

Figure 8 presents the results for the [19, 7, 3, 20] configuration. For this configuration results within the assumed unavailability ranges exist only for p_1 less than 3.5×10^{-2} . For $p_c = 8 \times 10^{-2}$, p_3 drops from 2×10^{-1} at $p_1 = 5 \times 10^{-3}$ to 6×10^{-2} at $p_1 = 3.5 \times 10^{-2}$. The black outlined diamond area corresponds to p_1 values of 3×10^{-2} .

Figure 7: Component unavailabilities for the detector configuration [20, 6, 3, 20]

Figure 8: Component unavailabilities for the detector configuration [19, 7, 3, 20]

For the [20, 7, 3, 20] configuration, results within the assumed unavailability ranges exist only for p_1 less than 4.5×10^{-2} . For $p_c = 8 \times 10^{-2}$, p_3 is above 2×10^{-1} at $p_1 = 5 \times 10^{-3}$ and drops to 1.7×10^{-2} as p_1 increases to 4.5×10^{-2} . For the [21, 7, 3, 20] configuration, results within the assumed unavailability ranges exist only for p_1 less than 5×10^{-2} . For $p_c = 8 \times 10^{-2}$, p_3 is above 3×10^{-1} for $p_1 = 5 \times 10^{-3}$ and reduces to 2.5×10^{-2} at $p_1 = 5 \times 10^{-2}$.

4.3. Discussion

The purpose of this section is to present ranges and combinations of possible component unavailability values that satisfy an overall system unavailability requirement. The unavailability of a component depends on its MTTF as well as on whether the component is repairable, and if yes, how often can the repair be undertaken and how long it lasts. At this phase of the telescope design it is not yet established whether all or which components will be

retrievable and repairable. For this reason, the present analysis provides an additional way to increase the system unavailability by increasing the number of deployed OMs.

Table 1 reports a representative selection of the results described in section 4.2 and presented in Figures 4 to 8, where

- The main cable unavailability is taken at 6×10^{-2} , 2×10^{-2} , 10^{-2} and 10^{-3} . According to the employed repair policy for the main cable, the respective MTTFs can be 1.3, 4.1, 8.3 and 83 years if immediate repair within one month MTTR is applicable (equation (36)) or 9.2, 28.7, 57.8 and 583 years if a fixed annual repair period of one month is applicable (equation (37)) etc.
- Two sets of values for the unavailabilities of the detector network cable and fibers are considered, namely (a) $p_1=5 \times 10^{-3}$, $p_2=2.5 \times 10^{-2}$, and (b) $p_1=1.5 \times 10^{-2}$, $p_2=7.5 \times 10^{-2}$. Assuming that all the components in the detector network are non-repairable, equation (35) provides the corresponding MTTFs for the (p_1, p_2) sets at 1000, 200 years for (a), and 330, 63 years for set (b).
- Indicatively, the cases discussed here consider that OMs and DUs have the same unavailabilities. Assuming that OMs and DUs are non-repairable, Table 1 provides the MTTFs required to satisfy the system unavailability target $U_T=10^{-1}$.

Table 1: Reliability requirements, in the form of MTTFs, for non-repairable OMs and DUs

main cable	detector network configuration & number of optical modules						
unavailability	[17,6,3,20]	[18,6,3,20]	[19,6,3,20]	[20,6,3,20]	[19,7,3,20]	[20,7,3,20]	[21,7,3,20]
p_c	6120	6480	6840	7200	7980	8400	8820
(a) minimum MTTF (in years) for $p_1=5 \times 10^{-3}$, $p_2=2.5 \times 10^{-3}$, $p_3=p_4$ and $U_T \leq 10^{-1}$							
6×10^{-2}	1160	150	82.5	57.5	36.2	30.2	26.2
2×10^{-2}	345	115	70.7	51.7	33.6	28.4	24.8
10^{-2}	305	111	69.2	50.9	33.2	28.2	24.6
10^{-3}	280	108	68.0	50.3	33.0	28.0	24.2
(b) minimum MTTF (in years) for $p_1=1.5 \times 10^{-2}$, $p_2=2.5 \times 10^{-3}$, $p_3=p_4$ and $U_T \leq 10^{-1}$							
6×10^{-2}	n.a.	n.a.	385	127	55.3	42.5	34.8
2×10^{-2}	n.a.	6000	185	93.5	47.7	37.9	31.7
10^{-2}	n.a.	2700	172	90.4	46.7	37.3	31.3
10^{-3}	n.a.	1900	163	88.0	46.0	36.9	30.9

In particular

- For $p_1=5\times10^{-3}$, $p_2=2.5\times10^{-2}$ and $p_c=6\times10^{-2}$, the base configuration [17, 6, 3, 20] requires MTTF of 1160 years for DUs and OM. If this MTTF is not technically achievable or is extremely expensive then additional redundancy in the form of extra DUs and OM is required. As the size of the network increases up to 8820 OM, the MTTF requirement for the DUs and the OM decreases down to 26.2 years. The MTTF requirements decrease also as the unavailability of the main cable decreases down to 10^{-3} (see Table 1 (a)).
- The importance of the main cable unavailability is reduced as the network redundancy increases. For example, assuming the base design featuring 6120 OM, as p_c drops from 6×10^{-2} to 10^{-3} the required MTTF for DUs and OM drops by a factor of 5, from 1160 to 280 years. For the configuration [19, 7, 3, 20] having 7980 OM, the MTTF for DUs and OM drops from 36 to 33 years, so the reduction due to the increased availability of the main cable is now only 3%.
- Similar behaviors are observed if the sector and branch unavailabilities increase to $p_1=1.5\times10^{-2}$ and $p_2=7.5\times10^{-2}$, respectively (see Table 1 (b)). In this case and for the p_c range considered here, the base detector network [17, 6, 3, 20] cannot meet the U_T target for any DU and OM MTTF. The [18, 6, 3, 20] network requires a $p_c\leq2\times10^{-2}$ and then the required MTTFs for OM and DU are in the order of a few thousand years.

These results indicate that the limiting constraint to the telescope unavailability will be set by the technically achievable MTTF and whether some or all of the detector network components may be retrievable and repairable. If repair of these components is not possible then higher numbers of OM must be deployed and the unavailability model may be used to suggest alternatives to meet the same overall unavailability target.

For example, assume that the manufacturer sets the maximum attainable MTTF for OM and DU at 150 years. The smallest network to satisfy the U_T target for $p_1=1.5\times10^{-2}$, $p_2=7.5\times10^{-2}$ and $p_c=6\times10^{-2}$ is now the [20, 6, 3, 20], featuring 18% redundancy compared to the base design. If the manufacturer resets the bound at 100 years, then the options in Table 1 are to apply the configuration [19, 7, 3, 20], or to use $p_c=2\times10^{-2}$. The latter can also be achieved by manipulating the factors affecting the reliability of the main cable, like distance to shore and routing, deployment and installation etc. Another alternative would be to consider maintenance for the OM and the DU. The unavailability of a component with 100

years MTTF and the option to be repaired annually for a fixed period of one month is 5.8×10^{-3} ; same as the unavailability of a non-repairable component with 860 years MTTF. In this case, smaller networks, less expensive to build but more expensive to maintain, like the [19, 6, 3, 20] with $p_c = 2 \times 10^{-2}$, become a feasible option and the choice depends on the project budget and money flow.

The above discussion demonstrates how the presented model can be used as a decision support tool for determining component reliabilities or adding redundancy to meet a certain overall unavailability requirement. At the end of the day, the optimal telescope configuration is the one that has the minimum total cost of investment, deployment, installation and maintenance during its operation. The choices on alternative network configurations, repair policies, cable routings etc need to consider the relationship between the prescribed component unavailabilities and the cost or the effort for achieving them.

5. CONCLUSIONS

This work presents a conceptual dependability analysis for a very large volume submarine neutrino telescope. The dependability criterion used here is quantitative unavailability, and an appropriate unavailability model is developed and presented here. The telescope unavailability is estimated based on the unavailabilities of the telescope components, namely the optical modules, the detection units where the optical modules are hosted, and the cable network transferring the measured neutrino signals to the facilities onshore.

The unavailability model is used to study the effects of different factors on the overall unavailability of the submarine system. Results are presented for different detector network configurations, with detector sizes ranging from 306 detection units of 6120 optical modules up to 441 detection units of 8820 optical modules.

Further research is under way to establish a realistic perspective on the practically attainable component reliability ranges, and develop suitable cost-reliability correlations to depict the trade-offs between component cost and reliability. These correlations will collaborate with the unavailability model developed here to enable a cost-based optimization towards optimal telescope configurations.

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APPENDIX – COMPONENT STEADY-STATE UNAVAILABILITY

This section presents an analysis deriving average steady-state unavailabilities for components that are testable and repairable. The derived steady-state unavailabilities will then be used in the relationships of section 3 to calculate the Detection Network unavailability. Note that regardless of the applicable repair regime, the following analysis shows that it is always possible to calculate the average availability of a component over a period of operation.

A.1. System periodically tested with constant repair time

Consider a system with constant testing period T , constant failure rate λ and deterministically known and constant repair duration d . An often used expression of steady state unavailability of such a system is obtained as follows:

The system unavailability p_c increases from zero as $1 - e^{-\lambda t}$, and reaches the value $1 - e^{-\lambda T}$ at the instance of the test or

$$p_c(t) = 1 - e^{-\lambda t} \quad \text{for } 0 \leq t \leq T$$

During the repair period, $T < t < T + d$, the probability that the system is unavailable, $p_c(t) = \Pr[U(t)]$, can be formulated as the joint probability of the “sure event” $\{A(T) + U(T)\}$ that the system is either available or unavailable at time T , and the event the system is unavailable at time t .

$$p_c(t) = \Pr[U(t)] = \Pr[\{A(T) + U(T)\}, U(t)]$$

Since the events “system available” $\{A(T)\}$ and “system unavailable” $\{U(T)\}$ are mutually exclusive it follows that

$$p_c(t) = \Pr[A(T), U(t)] + \Pr[U(T), U(t)]$$

and using conditional probabilities

$$p_c(t) = \Pr[U(t) | A(T)] \cdot \Pr[A(T)] + \Pr[U(t) | U(T)] \cdot \Pr[U(T)]$$

since $\Pr[U(t) | A(T)] = 1 - e^{-\lambda(t-T)}$, $\Pr[A(T)] = e^{-\lambda T}$, $\Pr[U(t) | U(T)] = 1$ and $\Pr[U(T)] = 1 - e^{-\lambda T}$, this becomes

$$\begin{aligned} p_c(t') &= (1 - e^{-\lambda(t'-T)}) \cdot e^{-\lambda T} + 1 \cdot (1 - e^{-\lambda T}) \\ &= (1 - e^{-\lambda(t'-T)}) \cdot e^{-\lambda T} + 1 - e^{-\lambda T} \quad \text{for } T \leq t' \leq T + d \end{aligned}$$

Setting $x = t' - T$ with $0 \leq x \leq d$ and noting that $1 - e^{-\lambda x} \cong 1 - 1 + \lambda x - \dots = \lambda x \leq \lambda d$, if (λd) is very small, that is if potential failures in the short interval of d can be ignored, then the term $(1 - e^{-\lambda(t'-T)}) \cdot e^{-\lambda T}$ in the equation above can be ignored and the system availability is given by

$$p_c(t) = \begin{cases} 1 - e^{-\lambda t} & \text{for } 0 \leq t \leq T \\ 1 - e^{-\lambda T} & \text{for } T \leq t \leq T + d \end{cases}$$

Assuming that the repair is perfect and hence when completed the unavailability drops to zero, the mean unavailability during the interval $0 \leq t \leq T + d$ is given by

$$\bar{p}_c = \frac{1}{T + d} \int_0^{T+d} p_c(t) dt$$

yielding

$$\bar{p}_c = \frac{1}{T + d} \left(\frac{\lambda(T + d) + (1 - \lambda d)e^{-\lambda T} - 1}{\lambda} \right) \quad (\text{A.1})$$

If T is such that $\lambda T \ll 1$ then $e^{-\lambda T} \approx 1 - \lambda T + \frac{(\lambda T)^2}{2}$ and $\frac{(\lambda T)^2}{2}(\lambda d) \approx 0$

Additionally, if $T \gg d$ then $\frac{T}{T + d} \approx 1$ and (A.1) reduces to ^[12–14]

$$\bar{p}_c = \frac{1}{2} \lambda T + \lambda d \quad (\text{A.2})$$

A more accurate analysis of periodically tested systems with constant time-to-repair is obtained if possible failures during the interval $(T, T + d)$ are taken into account. The state-transition diagram of this system is as shown in Figure 9.

Figure 9: Periodically tested component with constant repair time

The system exhibits a periodic behavior with period $T' = T + d$. Let $\pi_1(k)$, $\pi_2(k)$, $\pi_3(k)$ denote the state probabilities at the end of the k^{th} period. It is $\pi_3(k) = 0$ since, after the completion of the repair-time period d , the system, if under repair, transits from state 3 to state 1 with probability equal to unity. The state equations are

$$\begin{aligned}\dot{\pi}_1(t) &= -\lambda\pi_1(t) \\ \dot{\pi}_2(t) &= \lambda\pi_1(t) \\ \dot{\pi}_3(t) &= 0\end{aligned}\tag{A.3}$$

Consider the transitions of the system for the next period, i.e. between $k(T+d)$ and $(k+1)(T+d)$, and two intervals in this period: $0 < t < T$ and $T < t < T+d$.

In the first interval, $0 < t < T$, the state equations (A.3) are solved by applying Laplace transform with initial conditions $\pi_1(0) = \pi_1(k)$, $\pi_2(0) = \pi_2(k)$

$$\begin{aligned}\pi_1(t) &= \pi_1(k)e^{-\lambda t} \\ \pi_2(t) &= \pi_2(k) + \pi_1(k)(1 - e^{-\lambda t}) \quad \text{for } 0 < t < T \\ \pi_3(t) &= \pi_3(k) = 0\end{aligned}\tag{A.4}$$

For $t=T$, just before the test, equations (A.4) become

$$\begin{aligned}\pi_1(T) &= \pi_1(k)e^{-\lambda T} \\ \pi_2(T) &= \pi_2(k) + \pi_1(k)(1 - e^{-\lambda T}) \quad t=T \\ \pi_3(T) &= \pi_3(k) = 0\end{aligned}\tag{A.5}$$

For $t=T^+$, immediately after the test, equations (A.4) become

$$\begin{aligned}\pi_1(T^+) &= \pi_1(k)e^{-\lambda T} \\ \pi_2(T^+) &= 0 \\ \pi_3(T^+) &= \pi_2(k) + \pi_1(k)(1 - e^{-\lambda T})\end{aligned}\tag{A.6}$$

For the second interval, $T < t < T+d$, the state equations (A.3) are solved with $\pi_1(0) = \pi_1(T)$, $\pi_2(0) = 0$, $\pi_3(0) = \pi_3(T)$, using $x = t - T$

$$\begin{aligned}\pi_1(t) &= \pi_1(T)e^{-\lambda x} \\ \pi_2(t) &= (1 - e^{-\lambda x})\pi_1(T) \\ \pi_3(t) &= \pi_3(T)\end{aligned}\tag{A.7}$$

At the end of this period ($x=d$) these equations become

$$\begin{aligned}\pi_1(T+d) &= \pi_1(T)e^{-\lambda d} \\ \pi_2(T+d) &= (1 - e^{-\lambda d})\pi_1(T) \\ \pi_3(T+d) &= \pi_3(T)\end{aligned}\tag{A.8}$$

Immediately after the completion of the repair π_3 becomes zero and

$$\begin{aligned}
\pi_1(T+d) &= \pi_1(T)e^{-\lambda d} + \pi_3(T) \\
\pi_2(T+d) &= \pi_1(T)(1-e^{-\lambda d}) \\
\pi_3(T+d) &= 0
\end{aligned} \tag{A.9}$$

By virtue of (A.6) the above relationships give

$$\begin{aligned}
\pi_1(k+1) &= [1 - e^{-\lambda T} + e^{-\lambda(T+d)}] \pi_1(k) + \pi_2(k) \\
\pi_2(k+1) &= (e^{-\lambda T} - e^{-\lambda(T+d)}) \pi_1(k)
\end{aligned} \tag{A.10}$$

equations (A.10) provide recursively the state-probabilities $\pi_1(k+1)$, $\pi_2(k+1)$ after the completion of the $(k+1)^{\text{th}}$ test, in terms of $\pi_1(k)$ and $\pi_2(k)$. Given the initial conditions $\pi_1(0) = 1$, $\pi_2(0) = 0$, the equations can be solved for any value of k as

$$\begin{aligned}
\pi_1(k) &= \frac{b}{\alpha + b} + (1 - \alpha - b)^k \frac{\alpha}{\alpha + b} \\
\pi_2(k) &= \frac{\alpha}{\alpha + b} - (1 - \alpha - b)^k \frac{\alpha}{\alpha + b}
\end{aligned} \tag{A.11}$$

where $\alpha = e^{-\lambda T}(1 - e^{-\lambda d})$ and $b = 1$.

The unavailability of the system is the probability of occupying state 2 after the completion of the repair at the end of the k^{th} period of $(T+d)$, is therefore

$$p_c(k) = \frac{\alpha}{\alpha + b} - (1 - \alpha - b)^k \frac{\alpha}{\alpha + b} \tag{A.12}$$

Since $\lim_{k \rightarrow \infty} (1 - \alpha - b)^k = \lim_{k \rightarrow \infty} (e^{-\lambda T}(1 - e^{-\lambda d}))^k = 0$, equation (A.12) converges to an asymptotic steady-state solution. Therefore,

$$p_c(\infty) = \frac{e^{-\lambda T} - e^{-\lambda(T+d)}}{1 + e^{-\lambda T} - e^{-\lambda(T+d)}} \text{ and } A(\infty) = 1 - p_c(\infty) = \frac{1}{1 + e^{-\lambda T} - e^{-\lambda(T+d)}} \tag{A.13}$$

Note that, for $\lambda(T+d) \ll 1$, $e^{-\lambda T} \approx 1 - \lambda T$ and $e^{-\lambda(T+d)} \approx 1 - \lambda(T+d)$, (A.13) reduces to

$$p_c(\infty) = \frac{\lambda d}{1 + \lambda d} = \frac{d}{\frac{1}{\lambda} + d} \tag{A.14}$$

which is the steady-state unavailability for all types of failure and repair^[11].

Between successive periods (from $t=0$ until $t=T+d$) even when the steady-state has been achieved the unavailability changes with time, according to (A.4) for $0 < t < T$ and (A.7) for $T < t < T+d$.

The average unavailability is defined as

$$\bar{p}_{c,\text{int}}(k, k+1) \equiv \frac{1}{T+d} \int_0^{T+d} p_c(t) dt = \frac{1}{T+d} \left[\int_0^T p_c(t) dt + \int_T^{T+d} p_c(t) dt \right] \quad (\text{A.15})$$

equation (A.4) gives

$$\int_0^T p_c(t) dt = \pi_2(k)T + \pi_1(k) \left[T - \frac{1-e^{-\lambda T}}{\lambda} \right] = T - \pi_1(k) \frac{1-e^{-\lambda T}}{\lambda} \quad (\text{A.16})$$

while equation (A.7) gives

$$\int_T^{T+d} p_c(t) dt = d - \pi_1(k) \frac{e^{-\lambda T} - e^{-\lambda(T+d)}}{\lambda} \quad (\text{A.17})$$

Using (A.16) and (A.17), equation (A.15) becomes

$$\boxed{\bar{p}_{c,\text{int}}(k, k+1) = 1 - \pi_1(k) \frac{1 - e^{-\lambda(T+d)}}{\lambda(T+d)}} \quad (\text{A.18})$$

When the steady state is achieved, then $\pi_1(k) = \pi_1(\infty) = 1 - p_c(\infty)$ and by virtue of (A.13) it follows that

$$\boxed{\bar{p}_{c,\text{int}}(\infty) = 1 - \frac{1 - e^{-\lambda(T+d)}}{\lambda(T+d)(1 + e^{-\lambda T} - e^{-\lambda(T+d)})}} \quad (\text{A.19})$$

Note that, for $\lambda(T+d) \ll 1$ equation (A.19) is reduced to

$$\bar{p}_{c,\text{int}}(k, k+1) = 1 - \pi_1(k) \frac{\lambda(T+d)}{\lambda(T+d)} = 1 - \pi_1(k) = \pi_2(k)$$

This means that time dependence of the unavailability within the period is not important and $\bar{p}_{c,\text{int}}(k, k+1)$ is equal to the unavailability at the end of the k^{th} period $\pi_2(k)$.

An interesting special case is when the repair (of constant duration) can start immediately after the failure. This is equivalent to having $T \rightarrow 0$ and equation (A.19) yields

$$\bar{p}_{c,\text{int}}(\infty) = 1 - \frac{1 - e^{-\lambda d}}{\lambda d (2 - e^{-\lambda d})} \quad (\text{A.20})$$

and of course for $\lambda d \ll 1$ this reduces to

$$\bar{p}_{c,\text{int}}(\infty) = \frac{d}{\frac{1}{\lambda} + d} = \frac{d}{MTTF + d} \quad (\text{A.21})$$

This relationship is valid for any type of probability distribution functions for the times to failure and times to repair^[11].

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TABLE CAPTIONS

Table 1: Reliability requirements, in the form of MTTFs, for non-repairable OMs and DUs

FIGURE CAPTIONS

Figure 1: Overall neutrino telescope system

Figure 2: Three-dimensional detector network layout

Figure 3: Detector Network and Deep Sea Infrastructure Block Diagrams

Figure 4: Component unavailabilities for the detector configuration [17, 6, 3, 20]

Figure 5: Component unavailabilities for the detector configuration [18, 6, 3, 20]

Figure 6: Component unavailabilities for the detector configuration [19, 6, 3, 20]

Figure 7: Component unavailabilities for the detector configuration [20, 6, 3, 20]

Figure 8: Component unavailabilities for the detector configuration [19, 7, 3, 20]

Figure 9: Periodically tested component with constant repair time