THE EFFECT OF FUEL TEMPERATURE ON THE ESTIMATION
OF THE MODERATOR TEMPERATURE COEFFICIENT IN PWRs

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Abstract—The effect of fuel temperature on the estimation of the moderator temperature coefficient of reactivity (MTC) via noise analysis is investigated both theoretically and through numerical simulations. An analytical expression giving the envelop of the effect is provided. The simulations show that although for typical conditions the effect is small, it is characterized by a great deal of uncertainty and in certain occurrences it may become significant. The ranges of variation are quantified and the parametric effects are addressed. It is found that the induced bias is approximately uniform over a wide range of MTC values. This is supportive to the approach of calibrating the MTC estimates to the actual MTC values through a constant calibration factor.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( a_c )</td>
<td>moderator temperature coefficient of reactivity (MTC)</td>
</tr>
<tr>
<td>( a_r )</td>
<td>fuel temperature coefficient of reactivity</td>
</tr>
<tr>
<td>( b )</td>
<td>width of coolant channel</td>
</tr>
<tr>
<td>( C )</td>
<td>delayed neutron precursor number density</td>
</tr>
<tr>
<td>( C_{12} )</td>
<td>positive constant pertaining to coolant, ( cf. ) (1a)</td>
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<tr>
<td>( c )</td>
<td>specific heat capacity</td>
</tr>
<tr>
<td>( d )</td>
<td>width of fuel element</td>
</tr>
<tr>
<td>( E_0 )</td>
<td>energy released per fission</td>
</tr>
<tr>
<td>( E_i )</td>
<td>local (( i=1,2,3,4 ), or global (( i=s )) MTC estimators, ( E_i=</td>
</tr>
<tr>
<td>( F_{a1} )</td>
<td>positive constant pertaining to fuel, ( cf. ) (1b)</td>
</tr>
<tr>
<td>( F_{fb} )</td>
<td>flag to switch ‘on’ or ‘off’ power feed-back</td>
</tr>
<tr>
<td>( G_0 )</td>
<td>zero power transfer function</td>
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<tr>
<td>( H )</td>
<td>reactor height</td>
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<tr>
<td>( h )</td>
<td>heat transfer coefficient</td>
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<tr>
<td>( h_c )</td>
<td>hydraulic constant, ( h_c = (\rho_c c_b)^{-1} )</td>
</tr>
<tr>
<td>( K )</td>
<td>characteristic gain factor, ( cf. ) (23b)</td>
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<tr>
<td>( k_{\infty} )</td>
<td>infinite multiplication factor</td>
</tr>
<tr>
<td>( L )</td>
<td>diffusion length of thermal neutrons</td>
</tr>
<tr>
<td>( p )</td>
<td>resonance escape probability, ( p = \sum_{120}/\Sigma_{R,0} )</td>
</tr>
<tr>
<td>( Q_{sn} )</td>
<td>mean static heat flux</td>
</tr>
<tr>
<td>( R )</td>
<td>fundamental eigenfunction of Helmholtz equation</td>
</tr>
<tr>
<td>( r )</td>
<td>space variable</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature</td>
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1. INTRODUCTION

Monitoring of the moderator temperature coefficient of reactivity (MTC) during PWR core burn-up is an important operational requirement, connected with safety considerations. The MTC is defined as the change of reactivity per degree change of the core-averaged moderator (coolant) temperature. Estimation of the MTC via noise analysis has been contemplated as a desirable alternative to the standard boron dilution method, because noise techniques permit off-line monitoring and avoid interference with plant operation. However, limited success has been obtained. Noise measurements performed on power PWRs, gave estimations two to three times smaller than those of design predicted and/or measured by the boron dilution method (Oguma et al., 1995; Laggiard and Runkel, 1997; Pohlus, 1995; Herr and Thomas, 1991). Notwithstanding that, Oguma et al. (1995) and Herr & Thomas (1991) observed that the estimates were constantly proportional to the actual MTC values. On that basis, they matched successfully estimated MTCs during core burn-up, by calibrating the estimated magnitude to a known value of the MTC.

In a separate work, Antonopoulos-Domis and Housiadas (1999) have investigated theoretically and through numerical simulations the possibility of estimating a postulated MTC value via the processing of the commonly employed signals in noise measurements, namely, outlet coolant temperature fluctuations and in-core neutron...
density fluctuations. To eliminate uncertainties, they have neglected extraneous noises and fuel temperature effects in the analysis. The present work is an extension of this investigation, aiming at ascertaining the effect of fuel temperature on the MTC estimation.

As a rule, the MTC estimation is based on frequency-averaged spectral values at frequencies below 1 Hz, where a high coherence is usually observed between coolant temperature and neutron density, with the range of 0.1 to 0.6 Hz being the most representative in most experimental investigations (Oguma et al., 1995; Laggiard and Runkel, 1997; Pohlus, 1995; Herr and Thomas, 1991). In general, the fuel temperature effect is regarded as unimportant over the band 0.1 to 0.6 Hz, considering that the time constants of the heat transfer processes involved are of the order of few seconds (Laggiard and Runkel, 1997). However, there is much uncertainty with respect to the value of such parameters, hence a sensitivity analysis seems to be in order. Indeed, the results of the present work show that there may exist occurrences in which the MTC estimation is severely biased.

The present investigation evolves in the following way: In section 2, the bias introduced in the MTC estimation by fuel temperature is determined analytically with the help of an idealized reactor noise model. The analysis enables the identification of the parameters on which the effect is dependent, as well as the determination of an envelop for the estimation error. In section 3, a numerical model is implemented to generate noise data, which are used to quantify uncertainties and to assess separate effects. The conclusions are given in section 4.

2. THEORETICAL ANALYSIS

Consider a homogeneous critical reactor core, described by two group diffusion theory. Neutron noise is considered to be the result of random fluctuations in moderator and fuel temperatures. In a PWR core, the major effect arising from a change in moderator temperature is an opposite change in moderation, due to the effect of temperature on moderator density. On the other hand, a change in fuel temperature will lead to a similar change in resonance absorption by virtue of Doppler effect. Therefore, one may postulate that changes $\delta T_c$ and $\delta T_f$ induce respectively changes $\delta \Sigma_{12}$ and $\delta \Sigma_{a1}$ in cross-sections $\Sigma_{12}$ and $\Sigma_{a1}$, such that

$$\frac{\delta \Sigma_{12}(r,t)}{\Sigma_{120}} = -C_{12} \delta T_c(r,t) \quad (1a)$$

$$\frac{\delta \Sigma_{a1}(r,t)}{\Sigma_{a10}} = F_{a1} \delta T_f(r,t) \quad (1b)$$

In the above expressions, parameters $C_{12}$ and $F_{a1}$ are positive, they pertain respectively to coolant and fuel, and are assumed to be independent of space and time.

From first order multigroup perturbation theory, the reactivity fluctuations arising from (1a) and (1b) are

$$\delta \rho = - \int_\nu d^3 r \left( \Phi_{10} \Psi_{10} \right) \delta \Sigma_{a1} - \int_\nu d^3 r \left( \Phi_{10} \Psi_{10} - \Phi_{10} \Psi_{20} \right) \delta \Sigma_{12}$$

$$- \int_\nu d^3 r \Phi_{20} \Psi_{10} \nu \Sigma_f$$

(2)

where the product $\nu \Sigma_f$ has its usual meaning. For the case of a bare, critical reactor core, one may derive from (1a), (1b), and (2) that

$$\delta \rho(t) = - C_{12} (1-p) \cdot \int_\nu d^3 r \left( R^2(r) \delta T_c(r,t) \right)$$

$$- F_{a1} \Sigma_{R0} (1-p) \cdot \int_\nu d^3 r \left( R^2(r) \delta T_f(r,t) \right)$$

(3)

where $p=\Sigma_{120}/\Sigma_{R0}$, which represents the resonance escape probability, and $R(r)$ is the fundamental eigenfunction of the Helmholtz equation, satisfying the boundary conditions of zero flux at the extrapolated boundary of the core.
Consider now a one-dimensional reactor core of height $H$. For a large core, the extrapolated height is approximately equal to height $H$, hence $R(r) = \frac{2}{r} \sin \left( \frac{\pi Z}{H} \right)$, and following Fourier transform of (3), one obtains in the frequency domain

$$
\bar{\delta \rho}(\omega) = -\frac{2}{H} C_{12}(1-p) \cdot \int_0^H \sin^2 \left( \frac{\pi Z}{H} \right) \delta T_c(z,\omega) \, dz
$$

$$
- \frac{2}{H} F_{x1} \Sigma R \rho (1-p) \cdot \int_0^H \sin^2 \left( \frac{\pi Z}{H} \right) \delta T_f(z,\omega) \, dz.
$$

In point-kinetics, the reactivity is expressed by means of the core-average changes of $\delta T_c$ and $\delta T_f$ (Duderstadt and Hamilton, 1976)

$$
\bar{\delta \rho}(\omega) = -a_c < \delta T_c > -a_f < \delta T_f >
$$

where the core-average value $<F>$ of an axially dependent quantity $F(z)$ is

$$
<F> = \frac{1}{H} \int_0^H F(z) \, dz.
$$

In view of (4), (5), and (6), one may express the coefficients of reactivity $a_c$ and $a_f$ as following

$$
a_c = 2 C_{12}(1-p) \cdot \frac{<\sin^2 \left( \frac{\pi Z}{H} \right) \delta T_c(z,\omega)>}{<\delta T_c(z,\omega)>}.
$$

$$
a_f = 2 F_{x1} \Sigma R \rho (1-p) \cdot \frac{<\sin^2 \left( \frac{\pi Z}{H} \right) \delta T_f(z,\omega)>}{<\delta T_f(z,\omega)>}.
$$

As shown by Antonopoulos-Domis and Housiadas (1999) the magnitude of coefficient $a_c$ can be estimated from the global (point-kinetics) estimator $E=|G_{\Phi T}|/|G_0|$, where transfer function $G_{\Phi T}$ relates outlet coolant temperature fluctuations to the core-averaged flux fluctuations of the thermal neutrons, $G_{\Phi T} = \frac{<\delta \Phi_2>}{<\Phi_{2\omega}>}$, which can be further expressed as

$$
G_{\Phi T} = G_0 \frac{\bar{\delta \rho}}{\bar{\delta T}_{out}}.
$$

To explicate expressions (7a), (7b) and (8), solutions for $\delta T_c(z,\omega)$ and $\delta T_f(z,\omega)$ are required. By assuming a constant heat transfer coefficient along the channel, fuel and coolant temperatures $T_f(z,t)$ and $T_c(z,t)$ are determined from the following equations

$$
\rho_f c_f \frac{dT_f}{dt} = -\frac{h}{d} (T_f - T_c) + E_0 \nu \Sigma f \Phi_2
$$

$$
\rho_c c_c \frac{dT_c}{dt} + \rho_c c_c U \frac{dT_c}{dz} = \frac{h}{b} (T_f - T_c).
$$
By splitting all variables is static and fluctuating parts, and using the static equations, the following equations are obtained after linearization

\[ \tau_f \frac{d \delta T_f}{dt} = -\delta T_f + W \tau_c \delta \Phi_2 \]  
\[ \frac{d \delta T_c}{dt} + U_0 \frac{\partial \delta T_c}{\partial z} = -W \Phi_{20} \frac{\delta U}{U_0} + F_{fb} \frac{1}{\tau_c} (\delta T_f - \delta T_c) \]

where constant \( W \) is defined as \( W = E_0 d \nu \Sigma_f / b \rho c_c \), while fuel and coolant time constants \( \tau_f \) and \( \tau_c \) are defined respectively by \( \tau_f = \rho_f c_f d / h \), and \( \tau_c = \rho_c c_c b / h \). In (10b) \( F_{fb} \) is a flag which allows to accommodate, or eventually ignore, the feed-back effect of power fluctuations on coolant temperature.

As demonstrated by Antonopoulos-Domis and Housiadas (1999), open loop transfer functions are more suitable for performing MTC estimations. To obtain \( G_{\Phi T} \) as an open loop transfer function, equation (10b) will be solved by omitting the feed-back term, i.e. by setting flag \( F_{fb} = 0 \). Taking Fourier transform, along with specifying at the inlet \( \delta T_c (z=0, \omega) = \overline{\delta T}_{IN} (\omega) \), one obtains the following solution

\[ \overline{\delta T}_c (z, \omega) = \overline{\delta T}_{IN} (\omega) \exp \left(-i \omega t_R z / H\right) - \Delta T_{0m} \frac{\overline{\delta U}(\omega)}{U_0} \]

\[ \times \int_0^{z/H} \pi \sin (\pi z') \exp \left[-i \omega t_R (z / H - z')\right] dz' \]  

where \( \Delta T_{0m} = \overline{\delta T}_o (z=0) \), which represents the mean static temperature rise. At low frequencies, \( \omega t_R << 1 \), solution (11) can be approximated by the following expression

\[ \overline{\delta T}_c (z, \omega) \approx \overline{\delta T}_{IN} (\omega) - \Delta T_{0m} \frac{\overline{\delta U}(\omega)}{U_0} \left[1 - \cos \left(\frac{\pi z}{H}\right)\right] \]  

The preceding approximation enables the determination of the various space-dependent quantities required in the analysis. Specifically, by carrying out the integration dictated by (6), one may derive

\[ \overline{\delta T}_c \approx \frac{1}{2} \left( \overline{\delta T}_{IN} - \Delta T_{0m} \frac{\overline{\delta U}}{U_0} \right) \]  

\[ \overline{\delta T}_o \approx \overline{T}_{IN} - 2 \cdot \Delta T_{0m} \frac{\overline{\delta U}}{U_0} \]  

whereas, by setting \( \overline{\delta T}_{out}(\omega) = \overline{\delta T}_c (z = H, \omega) \), one obtains

\[ \overline{\delta T}_{out} = \overline{T}_{IN} - 2 \cdot \Delta T_{0m} \frac{\overline{\delta U}}{U_0} \]  

Next, apply the core average operator (6) on both sides of (10a), take Fourier transform, write \( \overline{\delta \Phi}_2 \approx \frac{1}{\beta} \overline{\Phi}_{20} \overline{\delta \rho} \) and replace \( \overline{\delta \rho} \) with expression (5). By this procedure, the low frequency solution for \( \overline{\delta T}_f \) is obtained as follows

\[ \overline{\delta T}_f \approx \frac{1 - h_c a_c \tau_c Q_{0m} / \beta}{1 + j \omega \tau_f + h_c a_c \tau_c Q_{0m} / \beta} \]
where $j = \sqrt{-1}$, $Q_{om}$ is the mean static heat flux, and $h_c$ is a hydraulic constant, defined by $h_c = (\rho_c c_v b)^{-1}$.

Expressions (7a), (13a) and (13b) indicate that at low frequencies

$$a_c = C_{12} (1- p).$$

(15)

Moreover, one may assume that at low frequencies

$$\frac{\sin^2 \left( \frac{\pi z}{H} \delta T_f \right)}{\delta T_f} \approx \frac{\sin^2 \left( \frac{\pi z}{H} \delta T_c \right)}{\delta T_c}.$$

(16)

The above approximation has been confirmed with the aid of the numerical model described in section 3 below. In view of (7b), (13a), (13b) and (16), the following result is obtained

$$a_f = F_{ai} \sum_{k=0} \rho(1- p).$$

(17)

Equations (15) and (17) indicate that at low frequencies, $\omega \tau_r << 1$, the coefficients of reactivity $a_c$ and $a_f$ behave effectively as constants. Now, using (5), (8), (13a), (13b) and (14), the transfer function $G_{\Phi T}$ can be written as

$$|G_{\Phi T}| = |a_c| |G_0| \left| \frac{\delta T_{IN} - \Delta T_{om}}{\delta T_{IN} - 2 \cdot \Delta T_{om}} \right| \frac{1 + a_f/a_c + j \omega \tau_f}{1 + j \omega \tau_f + h_c a_f \tau_c Q_{om} / \beta}. $$

(18)

If $a_f = 0$, the theoretical result of Antonopoulos-Domis and Housiadas (1999) is recovered, namely, that estimator $E = |G_{\Phi T}|/|G_0|$ yields $|a_c|$ when input noise is due to inlet coolant temperature fluctuations, and $|a_c|/2$ when noise is induced by coolant velocity fluctuations. Instead, if $a_f \neq 0$, estimator $E$ becomes biased. In this case, fuel temperature affects the estimation of MTC by the following factor

$$f = \left| \frac{1 + a_f/a_c + j \omega \tau_f}{1 + j \omega \tau_f + h_c a_f \tau_c Q_{om} / \beta} \right|. $$

(19)

The bias is frequency dependent, and does not depend on the nature of the input noise source. It may be positive, when $f > 1$, or negative, when $f < 1$. The associated relative percent error in estimating $|a_c|$, defined as

$$\varepsilon \% = 100 \times \frac{\text{ratio } E/|a_c| \text{ with fuel temperature bias} - \text{ratio } E/|a_c| \text{ without bias}}{\text{ratio } E/|a_c| \text{ without bias}}$$

(20)

is given by

$$\varepsilon \% = 100 \times (f - 1).$$

(21)

Equation (19) suggests that the effect of fuel temperature depends on several parameters, such as moderator temperature coefficient, coolant time constant, fuel time constant, power level, fuel composition (through the delayed neutron fraction $\beta$) and, obviously, fuel temperature coefficient. Table I provides possible ranges of variation for these parameters, along with a set of reference values (case 1). The data for coefficients $a_f$ and $a_c$ are based on theoretical and experimental evidence (Oguma et al., 1995; Herr and Thomas, 1991; Mosteller et al., 1991). For the time constants $\tau_c$ and $\tau_e$, the available data are poor. Generally, either time constants are considered to be of the order of few seconds (Holbert and Venkatesh, 1995). Owing to the large uncertainties associated with
these parameters, an extended range of 1 to 10 s is assumed in Table I. Regarding power, the range from 10% to 100% of the typical full power level is considered. Finally, for simplicity, fraction $\beta$ will be considered invariable.

Given the ranges of Table I, let us now determine a theoretical envelop for the error induced when estimating the MTC. The starting point is expression (19). Since the latter depends on too many variables, it is rearranged in the form

$$f = K \frac{|1 + j\omega \tau_p|}{|1 + j\omega \tau_p|}$$

(22)

so to group the variables into a smaller number, and bring out a characteristic time constant

$$\tau_p = \frac{\tau_f}{1 + h_{ar} \tau_c Q_{0m}/\beta}$$

(23a)

and a characteristic gain factor

$$K = \frac{1 + a_f/a_c}{1 + h_{ar} \tau_c Q_{0m}/\beta}.$$  

(23b)

The frequency-amplitude diagrams corresponding to (22) are shown in Figs. 1(a) and 1(b). The diagrams indicate that estimator E is biased high when $K>1$, biased low when $K<1$, and that the bias becomes manifest at frequencies (cyclic) $f \leq \max[f_1,f_2]$, where $f_1$ and $f_2$ are the characteristic frequencies $f_1 = 1/2\pi \tau_p$ and $f_2 = K/2\pi \tau_p$. Moreover, it is apparent that the effect becomes maximum when (i) $K$ becomes extreme (either maximum or minimum), and (ii) frequencies $f_1$, $f_2$ become maximum, so that the band of interest 0.1-0.6 Hz falls into the left part of the diagrams, i.e. within the maximum bias region. On the basis of (i) and (ii) above, one can easily identify the parametric combinations maximizing the effect. These combinations are given in Table I by cases 2 and 3, associated respectively, with the upper bound and the lower bound of the pursued error envelop. The corresponding error values are shown by the solid lines of Fig. 2. As can be seen, the envelop values are quite considerable. Note also that there exists a strong frequency dependence. Near 0.1 Hz the error may range between +42% and -68%, whereas near 0.6 Hz these values become respectively, +5% and -29%. In practical applications, average spectral values are considered. The mean error $\varepsilon_m$ over the frequency band 0.1-0.6 Hz is predicted to be included between an upper theoretical bound of +21% and a lower theoretical bound of -53%.

### 3. NUMERICAL RESULTS

The noise data required to substantiate the previously obtained theoretical results have been obtained with computer simulations. Numerical simulations are well suited to the objectives of the present work, since they offer the possibility of investigating the influence of parameters normally inaccessible to physical measurements, as for instance, the time constants $\tau_f$ and $\tau_c$. Moreover, they offer the possibility of obtaining data free of the various extraneous noises normally present in an operating PWR.
The reactor model used is an extension of the model employed in a similar numerical investigation (Antonopoulos-Domis and Housiadas, 1999). The model is based on a one-dimensional core description, two neutron energy groups, and one group of delayed neutrons. The neutron noise is induced by fluctuations $\delta \Sigma_{12}$ and $\delta \Sigma_{a1}$ of the cross-sections $\Sigma_{12}$ and $\Sigma_{a1}$, which in turn are driven by coolant and fuel temperature fluctuations according to (1a) and (1b). The temperature fluctuations $\delta T_f$ and $\delta T_c$ are inferred from a simplified thermal-hydraulic description, as given by equations (10a) and (10b) of section 2.

The linearized equations for the fluctuations of the neutron fluxes are the following

$$
\begin{align}
\frac{t_{TH}}{t} \frac{\partial \delta \Phi_1}{\partial t} &= \tau \frac{\partial^2 \delta \Phi_1}{\partial z^2} - \delta \Phi_1 - \left(1 - \frac{\Sigma_{12}}{\Sigma_{R0}}\right) \phi_{10} F_{a1} \delta T_f \\
&+ \frac{\Sigma_{12}}{\Sigma_{R0}} \phi_{10} C_{12} \delta T_c + (1-\beta) k = \frac{\Sigma_{a20}}{\Sigma_{120}} \Phi_1 - \frac{\lambda}{\Sigma_{R0}} \delta C \\
\frac{t_{D}}{t} \frac{\partial \delta \Phi_2}{\partial t} &= L^2 \frac{\partial^2 \delta \Phi_2}{\partial z^2} - \delta \Phi_2 + \frac{\Sigma_{120}}{\Sigma_{a20}} \delta \Phi_1 - \frac{\Sigma_{120}}{\Sigma_{a20}} \phi_{10} C_{12} \delta T_c 
\end{align}
$$

(24a)

(24b)

Fig. 1. Frequency-amplitude diagrams corresponding to Eq. (22) for $K>1$ (a), and $K<1$ (b).

Fig. 2. Error in the MTC estimation due to the effect of fuel temperature, as function of frequency.

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&+ \frac{\Sigma_{12}}{\Sigma_{R0}} \phi_{10} C_{12} \delta T_c + (1-\beta) k = \frac{\Sigma_{a20}}{\Sigma_{120}} \Phi_1 - \frac{\lambda}{\Sigma_{R0}} \delta C \\
\frac{t_{D}}{t} \frac{\partial \delta \Phi_2}{\partial t} &= L^2 \frac{\partial^2 \delta \Phi_2}{\partial z^2} - \delta \Phi_2 + \frac{\Sigma_{120}}{\Sigma_{a20}} \delta \Phi_1 - \frac{\Sigma_{120}}{\Sigma_{a20}} \phi_{10} C_{12} \delta T_c 
\end{align}
$$

(24a)

(24b)
The system of equations (24a)-(24c) together with equations (10a) and (10b) is solved numerically, by employing finite differences approximations to derivatives. The numerical code was verified by comparing with analytical solutions in the same way as in Antonopoulos-Domis and Housiadas (1999).

The simulations have been performed using the following reactor data, as typically encountered in power PWRs: core height of $H=3.6$ m, coolant transit time of $t_R=1$ s, diffusion length of $L=2.85$ cm, Fermi age of $\tau=27$ cm$^2$, diffusion time of $t_D=210$ ms, thermalization time of $t_{th}=5$ ms, delayed neutron fraction of $\beta=0.007$, and delayed neutron precursor decay constant of $\lambda=0.1$ s$^{-1}$. The input noise sources consisted of two uncorrelated pseudo-random number sequences, specified respectively, for the inlet coolant temperature fluctuations $\delta T_{IN}(t)$ and the coolant velocity fluctuations $\delta U(t)$. The two sequences are such that $\eta=2$, where $\eta$ is the ratio characterizing the input noise source, $\eta=\frac{\text{rms} (\delta U/U_o)}{\text{rms} (\delta T_{IN}/\Delta T_{0m})}$. In this way, a greater weight is attributed to coolant velocity fluctuations in inducing neutron and coolant temperature fluctuations, as indeed is observed experimentally (Shieh et al., 1987).

The parameters on which the fuel temperature effect is dependent, namely, the reactivity coefficients $\alpha_c$ and $\alpha_f$, the time constants $\tau_c$ and $\tau_f$, and the power level $Q_{0m}$, have been specified according to the data of Table I. The data corresponding to case 1 have been used to obtain reference predictions, while those of cases 2 and 3 have been employed to determine the limits of variation. The moderator temperature coefficient $\alpha_c$ and the fuel temperature coefficient $\alpha_f$ are postulated via parameters $C_{12}$ and $F_{a1}$, on the basis of Eqs. (15) and (17). The postulated MTC value is to be recovered by the processing of the signals produced by the model, considered here as experimental signals. In line with the standard practice employed in noise experimental investigations, the following signals are considered: coolant outlet temperature fluctuations $\delta T_{ou}$, and thermal neutron flux fluctuations $\delta \phi_i$ from the in-core detectors. As suggested by Antonopoulos-Domis and Housiadas (1999), the magnitude of the MTC is estimated, either from the local neutron signals $\delta \phi_1$, $\delta \phi_2$, $\delta \phi_3$, $\delta \phi_4$ (from the detectors positioned respectively, at $z_1=H/5$, $z_2=2H/5$, $z_3=3H/5$, and $z_4=4H/5$) or from the sum of all neutron signals $\delta \phi_s = \sum_{i=1}^{4} \delta \phi_i$, using global estimator $E_s$ (all estimators $E_i$, $i=1, 2, 3, 4, s$ are defined in the Nomenclature). Since open loop transfer functions give better estimations (Antonopoulos-Domis and Housiadas, 1999), the simulations were forced to produce open loop results, by setting flag $F_{fb}=0$ in Eq. (10b).

In Fig. 2, superimposed on the theoretically derived error envelop, are shown the errors inferred from the numerical simulations. The error values are deduced by comparing the MTC estimations from a simulation performed with $\alpha_f=0$ and a simulation performed with the reference parametric values of Table I (case 1). The vertical bars of Fig. 2 illustrate possible variations in the predictions, arising from variations in the values of the parameters. They have been determined using the envelop parametric values of cases 2 and 3 of Table I. Two estimators have been used. The global estimator $E_s$, and the local estimator $E_4$, which uses the neutron signal from the detector at $z_4=4H/5$, positioned closer to the outlet. The results of Fig. 2 indicate that with reference parametric values the fuel temperature effect remains very small over the frequency range considered. However, it is very sensitive to the values of the parameters, and there exist occurrences for which it becomes considerable. The frequency-averaged error $\epsilon_m$ is predicted to range within the following limits: $-37\% \leq \epsilon_m \leq +12\%$. One may notice that the simulation results fall well within the theoretical limits of variation, thereby confirming that the latter can be taken as an effective error envelop. Also notice that the bias is practically the same, regardless of employing a local or global estimator in estimating the MTC magnitude.

Figures 3(a)-(e) illustrate the influence of each parameter separately. The solid circles show the frequency-averaged error $\epsilon_m$, as predicted from simulations in which a single parameter changes and the others are kept at their reference values. The variances due to variation in the parameters are shown with vertical bars. The solid lines show the theoretical envelopes of variation, as evaluated with the help of (19) and (21).

Figure 3(a) shows that, as expected, the fuel temperature effect may become important only when the corresponding coefficient of reactivity $\alpha_f$ is large. Instead, if the value of $\alpha_f$ is less than $1.5 \cdot 10^5$ K$^{-1}$, the effect will be always small (less than 10%), for any combination of parameters. Similarly, as depicted by Fig. 3(b), the fuel temperature effect may become of concern, only when the fuel time constant is small, for instance less than 4 s. For $\tau_f>4$ s, the effect can be neglected, regardless of the value of the other parameters.
Fig. 3. Frequency-averaged error in the MTC estimation due to fuel temperature effects: the influence of each parameter.
The common trend observed in Figs. 3(c) and (d) is that by increasing either the power level, or the coolant time constant, the bias is magnified. Under reference conditions the influence of either parameters is negligible, but the associated uncertainties are large. In particular, as can be seen in Fig. 3(c), a rise in power may be accompanied by a serious underestimation of the MTC if the other parameters have values close to the lower envelop case (case 3 of Table I). For the same set of parameters, and with a large coolant time constant, the results of Fig. 3(d) indicate that the MTC may be substantially underestimated.

Figure 3(e) shows the effect of the MTC magnitude itself. Here again, one observes that with reference parameters the effect is negligible, but it may result in substantial underestimation of the MTC if combined with extreme parametric values. As can be seen, the mean error $\epsilon_m$ can be approximately considered as uniform over the whole range of the MTC values, provided that the other parameter are not changing. This finding is supportive to the experimental approach of calibrating the MTC estimates to the actual MTC values by means of a constant calibration factor (Oguma et al., 1995; Herr and Thomas, 1991).

4. CONCLUSIONS

The present investigation addressed the effect of fuel temperature on the estimation of the moderator temperature coefficient of reactivity (MTC) via noise analysis. The effect was investigated, both theoretically and through numerical simulations, in the frequency band of 0.1-0.6 Hz, which is the one typically employed in MTC estimations. It was found that:

1. The primary parameters on which the fuel temperature effect is dependent are, as expected, the following: the fuel temperature coefficient of reactivity $a_f$, the fuel time constant $\tau_f$, the coolant time constant $\tau_c$, the MTC magnitude $a_c$, and the power level. An analytical expression has been derived, providing the envelop of the effect [cf. Eq. (19)].
2. If the parameters above are at their reference values (case 1 of Table I), the produced effect can be ignored. However, the effect is characterized by a great deal of uncertainty due to large possible variations in the parameters. In certain occurrences the effect may become considerable. The simulations indicated that the MTC magnitude may be underestimated by as much as about -40%, or eventually overestimated by approximately +10%. The conditions associated with significant estimation errors are identified by cases 2 and 3 of Table I.
3. The effect can be always ignored if $\tau_f > 4$ s, or $|a_f| < 1.5 \cdot 10^{-5}$ K$^{-1}$, no matter what are the values of the other parameters.
4. The effect is practically the same, independently of using a local or global estimator to estimate the MTC magnitude.
5. By changing the MTC value alone, the induced bias remains approximately uniform. This result reinforces the experimentally suggested approach of calibrating the MTC estimates to the actual MTC values by means of a constant calibration factor.

REFERENCES